Mathematics for Systems Biology and Bioinformatics

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Exercise sheet no. 7

Submission until 12.12.2012 10:00 am in the tutorials

Homework 12: Transformation Matrix (10 Points)

Linear mappings can be described by a transformation matrix A via matrix multiplication such that the vector x maps on y:

$$y = A\,x \qquad \Leftrightarrow \qquad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{11} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{1}$$

a) Use the vector $x_0 = (1,2)^T$ calculate $y = (y_1, y_2)^T$ for each transformation matrix:

i.) $A = \begin{pmatrix} \cos 45° & -\sin 45° \\ \sin 45° & \cos 45° \end{pmatrix}$ iii.) $C = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ v.) $E = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

ii.) $B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ iv.) $D = \begin{pmatrix} 3 & 0 \\ 0 & 1/3 \end{pmatrix}$ vi.) $F = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

b) Draw the results in a coordinate system and identify the matrices with the mappings:
scaling, projection, rotation counterclockwise, squeeze mapping, reflection, horizontal sheer mapping

c) Use the above types of matrices to rotate the vector $x = (-1,-1)^T$ by 30° clockwise. Then scale the resulting vector by factor 1.5 using a suitable transformation matrix.

Find the transformation matrix for 'rotate 30° clockwise and scale with factor $-\sqrt{3}$'.
Test the result with $x = (1, 1/\sqrt{3})^T$ and draw a sketch of it.

d) Transform the 3 vectors x_1, x_2, x_3 using matrix G and draw them in a coordinate system

$$G = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \quad x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ x_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ x_3 = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

Why is this a special case?

e) Calculate the eigenvalues of the matrix G using the characteristic polynom

$$det(\lambda \mathbb{1} - G) = 0$$

For each eigenvalue λ, find one eigenvector.

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